Teacher Guide for Fibonacci Numbers

Special notes and timing

There are two versions of this lesson—one is the full set, and the other is an abbreviated worksheet that covers the material from lessons 1, 4, 6, and 7 of the full set.

Learning Objectives

Students will:

- Describe and apply the rule for making the Fibonacci numbers
- Recognize patterns in the Fibonacci numbers
- Describe how the golden ratio is related to the Fibonacci numbers
- Describe similarities and differences between the Fibonacci numbers and the Lucas numbers
- Create new number patterns

Materials

- Math journal or notebook (all 10 lessons)
- Calculator (lessons 3, 6, 7, and 9), optional
- Master for Fibonacci Lesson 4
- Ruler (lesson 7)
- Miscellaneous boxes or cans--for example, cereal or cake mix boxes of different sizes, soup and vegetable cans of different sizes (lesson 7)
- Graph paper (1-2 sheets per student) (lesson 8)

Helping Questions

How might you begin?
What pattern do you see?
What is the next step (in the process, in the pattern)? Does that help you see any patterns?
Compare your results with your list of Fibonacci numbers. Do you notice any relationships? (Ask about Lucas numbers or their own number sequences in Lessons 9-10.)
What do you notice? What else do you notice?
Have you asked one of your classmates if he or she could see a pattern?
Why do you think that happens? (*Students may not know the answer to this question, but it helps them to think about it and try to explain it.*)
What did you try that didn't work? Can you learn something from that?
Can you explain it in a different way?
For lesson 10: What other kinds of rules could you use to create a number sequence? (In the Fibonacci numbers, you added two numbers in a row. What else could you do?)

Assessment Options

- Look at the students' math journals. Ensure that they write down enough details so that they could look back and understand their work without having the lesson cards available.
- Students should be able to compute patterns with Fibonacci numbers as well as explain them verbally.
- In Lesson 10, students are asked to create their own sequence. Look at how many different experiments the students tried with their sequence.

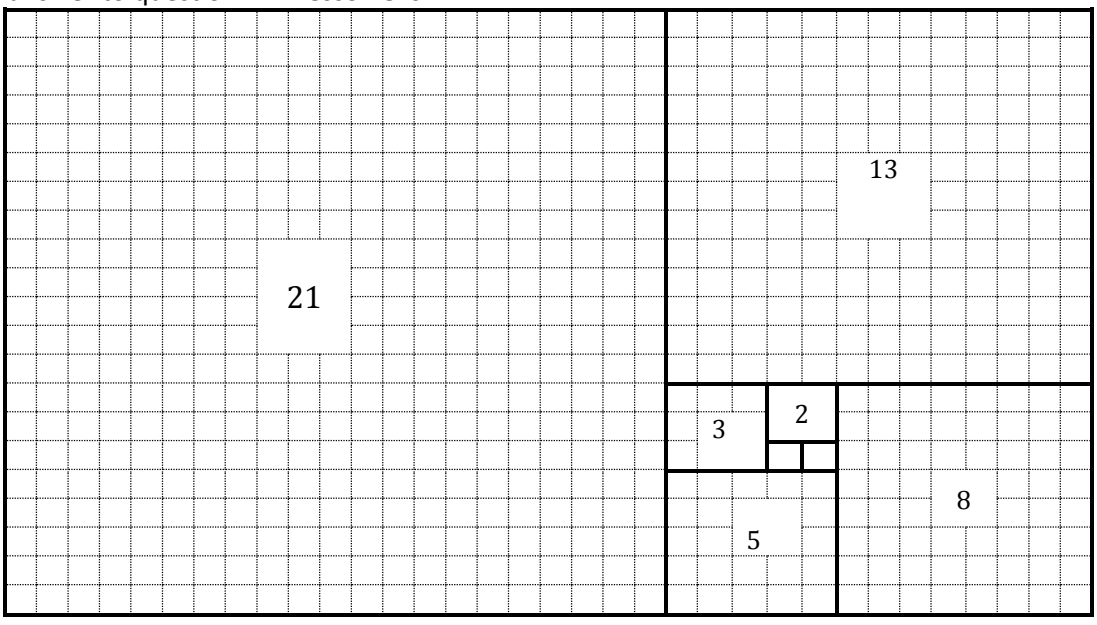
Mathematical Notes

In the section about rows of squares in Lesson 4 (lesson 2 in the worksheet version), students are quite tempted to say that since 3 squares can be covered in 3 different ways, then it must be that 4 squares are covered in 4 ways and 5 squares are covered in 5 ways. Help them see that this isn't quite the right conjecture. Another common (incorrect) conjecture is that the 6 squares can be covered in 12 different ways. It's fine to let them have this conjecture and then discover the error themselves—but make sure they look critically at their coverings of the squares to make sure that they have found them all!

There are some YouTube videos about math in nature. Unfortunately, the ones with the best math are also based upon the premise that the creator doodles in her math class instead of paying attention. However, the mathematics is sound—preview these before showing to students to see how much you object to the doodling in math class (or skip over the first part). Search on "doodling in math spirals" in YouTube, or go to http://www.youtube.com/watch?feature=endscreen&NR=1&v=ahXIMUkSXX0 (part 1, introduction to Fibonacci numbers and spirals), http://www.youtube.com/watch?v=lOIP_Z_-0Hs (part 2, which addresses plant growth and the golden ratio very nicely), and http://www.youtube.com/watch?v=14-NdQwKz9w (part 3, which discusses Lucas numbers and why plants actually develop Fibonacci numbers). The three parts must be watched in order if you really want to understand them.

There's a nice part about the golden ratio in the movie *Donald Duck and Mathmagic Land*. You can find it on YouTube at http://www.youtube.com/watch?v=ReJOK8RMzPE, or by searching YouTube for the title "Donal Duck in Mathmagic Land (Golden Rectangle)." (Yes, the misspelling of Donald is supposed to be that way.)

The answer to question 1 in Lesson 8 is:



If you want a counterpoint to all of the information about the internet about the golden ratio and the Fibonacci numbers, the video that is Keith Devlin's speech to the Museum of Mathematics about the myth of the golden ratio. This video is at http://www.youtube.com/watch?v=JuGT1aZkPQ0, or by

searching for "Math Encounters – Fibonacci & the Golden Ratio Exposed." There's a very long build-up—start watching the video about 37 minutes into it.
Guide to the video:

- *Minutes 37-60 tell about all the things that are wrong about popular notions about the golden ratio, but starting at the 1-hour point, there is a list quite a few ways that the golden ratio shows up naturally, most notably in mathematics.*
- *At 1:10 is the start of the list of things that really are described by the Fibonacci numbers—the part at 1:14 is particularly interesting, because it refers to the notion that the Fibonacci numbers show up in nature in numbers of flower petals or the number of spirals on a pine cone.*
- *At 1:20 there is a good description about the leaves spiraling around the stem of a plant.*
- *At 1:24 there is an explanation of why the Fibonacci numbers appear in nature.*
- *Around 1:27 there is a lot of discussion about irrational numbers that the students haven't yet learned, but it gets explained in terms of a continued fraction, which the students can probably understand.*

Extensions

Students are asked in Lesson 10 to create their own number pattern. Have students look up their number pattern on the Online Encyclopedia of Integer Sequences (https://oeis.org/). It is likely that most of the results will have explanations beyond the students' ability (and perhaps beyond the ability of many professional mathematicians!), but the students may enjoy learning that their sequence is a known sequence, or that they have found something new.

Lessons 7 and 10 provides several extensions for student work through research. A good website for mathematics history is the MacTutor History of Mathematics Archive: http://www-history.mcs.st-andrews.ac.uk/index.html.

Have students search for "Fibonacci numbers in nature" on the internet or in books. There will probably be results about flowers, pineapples, and pine cones. They can bring in items from home to explore the Fibonacci numbers in nature.

You could have students watch the Keith Devlin video and write a position paper on whether all of the information on the internet about the golden ratio and the Fibonacci numbers is true, or ask them to research the writings that are referenced in the talk (specifically, the books *The Golden Ratio*, by Mario Livio, and *The Man of Numbers*, by Keith Devlin). This video is at http://www.youtube.com/watch?v=JuGT1aZkPQ0, or by searching for "Math Encounters – Fibonacci & the Golden Ratio Exposed." (More details about the video are in the "Mathematical Notes" section.)

Teacher Reflection

- Which students were able to see the patterns quickly, and which students struggled to see the patterns?
- Did some students see the algebraic patterns more quickly than the visual patterns, or vice versa? What extensions might be appropriate for those students next? What support can you provide for their areas of weakness?
- Did some students get frustrated if they couldn't find a pattern immediately? What can you do to ease that frustration?
- Did some students work better individually or in pairs/small groups? What support can you provide to the students to work outside their comfort zone?

- What were the greatest challenges for the students?

Standards Addressed

Common Core State Standards (and Colorado Academic Standards in Mathematics)
1. Number Sense, Properties, and Operations
2. Patterns, Functions, and Algebraic Structures
4. Shape, Dimension, and Geometric Relationships (lessons 7-8)

NCTM (National Council of Teachers of Mathematics) Content Standards
Algebra
Geometry
Measurement

NCTM Process Standards
Problem Solving
Reasoning and Proof
Communication
Connections
Representation

References Used

Charles D. Miller, Vern E. Heeren, and John Horsby, *Mathematical ideas*, 9th edition. Addison Wesley, Boston, 2001.

Arthur T. Benjamin and Jennifer J. Quinn, *Proofs that really count: The art of combinatorial proof*. Dolciani Mathematical Expositions, v. 27, Mathematical Association of America, Washington, DC, 2003.

This unit was written by Beth Schaubroeck, U.S. Air Force Academy, in conjunction with Colorado Springs School District 11. Please direct questions and comments to beth.schaubroeck@usafa.edu.