Fibonacci Numbers

Lesson 1 of 4

In 1202, the mathematician Leonardo Pisano Fibonacci (pronounced fi-buh-NAH-chee) published a book with the famous Fibonacci sequence in it. (A sequence is a list of numbers.)

This sequence starts with the numbers 1 and 1.

To get the next number, you add the first two: $1 + 1 = 2$.
Now the sequence looks like 1, 1, 2.

To get the next number, you add the previous two numbers: $1 + 2 = 3$.
Now the sequence looks like 1, 1, 2, 3.

Keep doing this to build the sequence. The next two numbers in the sequence are 5 (from $2 + 3 = 5$) and 8 (from $3 + 5 = 8$).
Now the sequence looks like 1, 1, 2, 3, 5, 8.

1. Write down the first part of the sequence, shown below, and then fill in the blanks for the next 6 Fibonacci numbers.
1, 1, 2, 3, 5, 8, ___, ___, ___, ___, ___, ___

2. In the first few terms (numbers) of the sequence, you can see a pattern of whether the numbers are even or odd. Fill in the table below to see the pattern.

Number	Even or odd
1	odd
1	odd
2	even
3	
5	
8	

3. Write 1-2 sentences to describe the pattern and explain why it happens. (Hint: What is an odd number plus an odd number?)

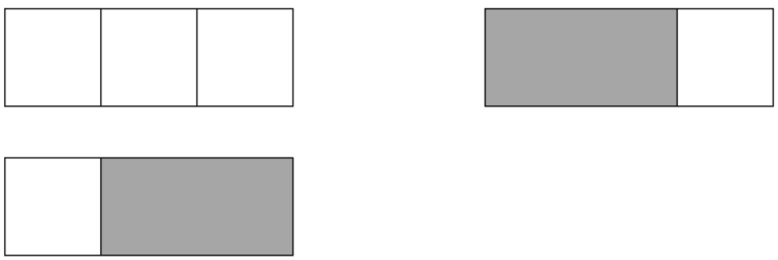
Fibonacci Numbers

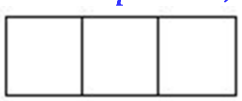
Lesson 2 of 4

We are going to explore how many different ways you can cover a length of squares with either squares or simple rectangles.

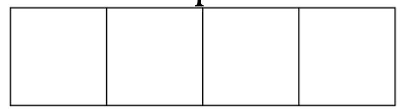
First, cut out the squares and rectangles from the master that goes with this lesson.

Next, look at the grid of 3 squares. How many different ways can you cover that grid with the squares and rectangles? Record all the possibilities on the accompanying sheet. Your answer should look like this:

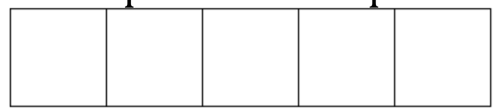


Notice that there are 3 ways to do it, because we count  *and*  *as different ways to cover the three squares, or you can just use 3 small squares*  *.*

1. Now figure out how many ways there are to cover the row of 4 squares. Record all of the different pictures on the paper of squares.



2. Repeat with 5 squares.



3. Fill in the table below.

Row of ___ squares	Number of ways to cover with squares and rectangles
3	*3*
4	
5	

4. **Predict** Look back at the list of Fibonacci numbers you made on Lesson 1. What do you think will be the next row in the table?

5. **Check** Use the row of 6 squares to see if your prediction is correct.

Fibonacci Numbers

Lesson 3 of 4

Remember that the Fibonacci numbers are
1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, etc.
Now let's look at another pattern in the Fibonacci numbers.

1. Find the quotients, using long division or a calculator. Record the answer up through the thousandths place. Record the problem and the answer in your math journal.

1 ÷ 1= __________

2 ÷ 1 = __________

3 ÷ 2 = __________

5 ÷ 3 = __________

8 ÷ 5 = __________

2. Look at the list of Fibonacci numbers you made on Lesson 1, and at the pattern of divisions done in problem 1. Figure out the pattern, and fill in the next several pieces of the pattern. Record your answers below.

13 ÷ 8 = 1.625

____ ÷ ____ = __________

____ ÷ ____ = __________

____ ÷ ____ = __________

____ ÷ ____ = __________

____ ÷ ____ = __________

3. **Examine** Now look at the list of answers you got for problems 1 and 2. Are the answers getting close to some number?

Fibonacci Numbers and the Golden Ratio

Lesson 4 of 4

Look at the answer to Question 3 of Lesson 3 before you read further.

If you keep doing divisions of consecutive (one right after the other) Fibonacci numbers, like you did in Lesson 3, you will get closer and closer to a special number called the **golden ratio**. The golden ratio shows up in art, architecture, music, and nature. The ancient Greeks thought that rectangles whose sides form a golden ratio were pleasing to look at.

The golden ratio is often symbolized by the Greek letter phi, which looks like this: ϕ. It is the number ϕ = 1.6180339887… (and so on). When you write out the decimal, it continues on forever without repeating or making a pattern.

1. Collect boxes and cans from home, or use some that are in the classroom. For boxes, measure the height and width of the boxes, and for cans, measure the height and diameter. Make a table of what you measured.

Description	Height in cm	Width in cm	longer one ÷ shorter one
Cereal box	*30.0*	*19.4*	*1.546*

2. **Examine** Is the quotient close to the golden ratio ϕ? It probably will never exactly equal ϕ, but many of the items are probably close. Which ones are closest?

3. **Explain** Why do you think that the common household items show the golden ratio?