

# Iteration

*Lesson 1 of 6, work individually or in pairs*

This set of lessons will help you learn about iteration (it-er-A-shun). This is where you repeat a process, and the answer to the first step is the starting point for the next step.

We will start with the **hailstone numbers**. Here's how we find hailstone numbers. Start with any number you like. At each step, follow these rules:

- If it is even, divide it by 2.
- If it is odd, multiply it by 3 and then add 1.
- Use the answer as the new starting point and repeat the rules.

For example, if we start with 13, we do these steps:

13 is odd, and  $13 \times 3 = 39$  and  $39 + 1 = 40$

40 is even, and  $40 \div 2 = 20$

20 is even, and  $20 \div 2 = 10$

10 is even, and  $10 \div 2 = 5$

5 is odd, and  $5 \times 3 = 15$  and  $15 + 1 = 16$

16 is even, and  $16 \div 2 = 8$

8 is even, and  $8 \div 2 = 4$

4 is even, and  $4 \div 2 = 2$

2 is even, and  $2 \div 2 = 1$

1 is odd, and  $1 \times 3 = 3$  and  $3 + 1 = 4$

*Materials Needed: Pencil  
Math Journal or Notebook*

**Now we realize that the numbers will be stuck in the pattern 4, 2, 1, 4, 2, 1, forever.**

1. Write down the hailstone numbers, starting with the number 21. Keep going until you get to the 4, 2, 1 pattern.
2. Write down the hailstone numbers, starting with the number 6. Keep going until you get to the 4, 2, 1 pattern.
3. Write down the hailstone numbers, starting with the number 11. Keep going until you get to the 4, 2, 1 pattern. (Hint: Do you eventually get to numbers that you have seen in another problem or example?)
4. **Challenge Question** Write down the hailstone numbers, starting with the number 18. Keep going until you get to the 4, 2, 1 pattern.

*Standards: Patterns*

# Iteration

Lesson 2 of 6, work individually or in pairs

**Review** Look back at your work from lesson 1 and the rules about how to make hailstone numbers. Remember that at each step, follow this rule:

- If it is even, divide it by 2.
- If it is odd, multiply it by 3 and then add 1.
- Use the answer as the new starting point and repeat the rules.

In weather, if hail falls from the sky, it is formed by a drop of rain being bounced up and down in the clouds before falling down to earth.

The hailstone numbers seem to bounce up and down just like the hail that falls from the sky does.

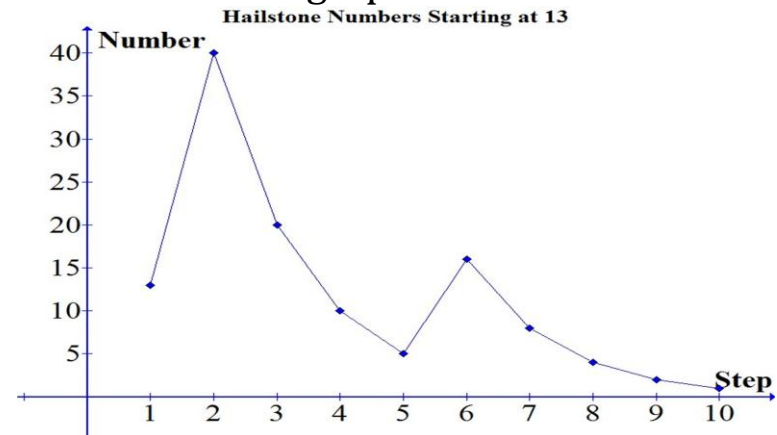
If we graph the hailstone numbers, we can see this. Remember that in lesson 1, the example used the number 13.

*Materials Needed: Pencil  
Math Journal or Notebook*

Our answers looked like this:

Step	Number
1	13
2	40
3	20
4	10
5	5
6	16
7	8
8	4
9	2
10	1

We will make a line graph with what step we took on the horizontal axis and the number on the vertical axis. The line graph looks like this.



1. Create a line graph similar to the one above for all of the exercises in Lesson 1.

*Standards: Patterns, graphs*

# Iteration

Lesson 3 of 6, work individually or in pairs

Another way we can iterate or repeat a process is with pictures instead of numbers. Here we will start to make a *Sierpinski triangle*, named after Waclaw Sierpinski, who described it in 1916.

Use triangle graph paper or measure carefully

1. Draw and cut out an equilateral triangle. If you are using the triangle graph paper, make sure there are 16 little triangle sides on each side of your triangle.
2. Find the center of each side of the triangle, and mark it with a pencil. Use a pencil and ruler to connect the center points. Now cut along your drawn pencil lines. Keep the three corner triangles, and remove the center triangle. It should look like this, if the blue is the paper that remains:

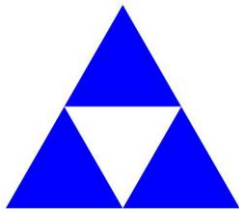
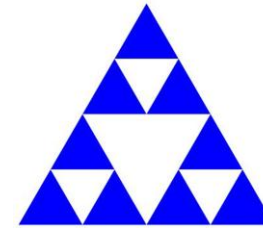


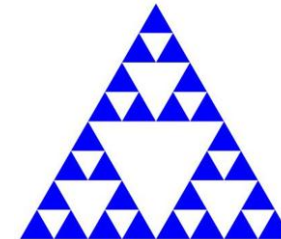
Image credit: "Sierpinski Sieve" from the Wolfram Demonstrations Project <http://demonstrations.wolfram.com/SierpinskiSieve/>, contributed by Peter House

*Materials Needed: Pencil, Scissors, and Tape  
Math Journal or Notebook  
Triangle Graph Paper*

3. Now with each smaller triangle that is left, find the center of each side of the triangle, and mark it with a pencil. Use a pencil and ruler to connect the center points. Now cut along your drawn pencil lines. Keep the three corner triangles, and remove the center triangle. Do this with all three of the triangles you had left after question 2. Now what you have left should look like this:



4. You should be seeing a pattern by now! Repeat that pattern with the triangles you have left (or re-read question 3 and repeat the steps). When you are done, it should look like this:



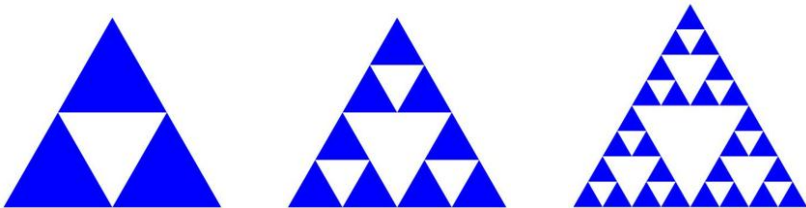
**Tape or glue the triangles to the paper, and save them for the next lesson!**

*Standards: Geometry, patterns*

# Iteration

Lesson 4 of 6, work individually or in pairs

Look again at the different steps of the Sierpinski triangle in lesson 3. You should have completed these steps:



1. For each step, fill in the table below. Some of it has been filled in for you.

Step	Number of <u>new</u> white triangles	Total number of white triangles	Total number of blue triangles	Total number of triangles (white + blue)
1	1	1	3	4
2	3	4		
3				

Materials Needed: Pencil  
Math Journal or Notebook

2. **Predict** What will the entries in the table be for Step 4? Write a sentence or two to explain why the numbers are what they are. (Hint: Do some numbers from step 2 get repeated in step 3?)

3. **Check** Draw new triangles on your last picture, and check to see if your prediction in Question 2 was correct.

To make a Sierpinski triangle, you would continue the process of cutting out center triangles forever. It would look something like this:

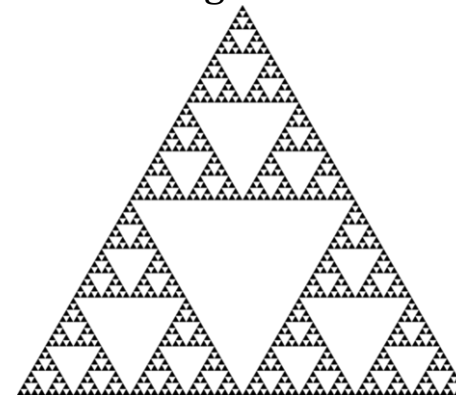


Image credit: "Sierpinski Triangle" from Wikimedia Commons, [http://en.wikipedia.org/wiki/File:Sierpinski\\_Triangle.svg](http://en.wikipedia.org/wiki/File:Sierpinski_Triangle.svg), contributed by user Marco Polo.

Standards: Geometry, patterns


# Iteration

*Lesson 5 of 6, work individually or in pairs*

In this lesson, you will do another kind of iteration with triangles to make the *Koch snowflake*. The Koch snowflake is named after the Swedish mathematician Niels Fabian Helge von Koch (pronounced “Kahkh”). Triangle graph paper may be helpful, or you can measure carefully.

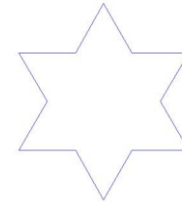
1. Use a pencil (not a pen), make an equilateral triangle in the middle of the page. One that is 9 units along each side works well.

2. Erase the center third of each side of your triangle (if your triangle is 9 units per side, erase 3 units in the center of each side). Replace the

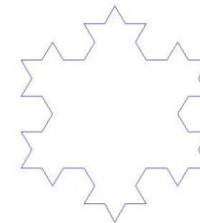
erased part with a tent . Each side of the tent is the same length as the line segment you erased.

*Materials Needed: Pencil  
Math Journal or Notebook  
Triangle Graph Paper*

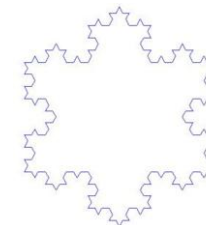
Now you should have something that looks like this.



3. Repeat the process—on every straight side, erase the center third, and replace that center third with a tent. You should have something like this:



4. Do you see the pattern? Repeat again to get this picture.



To make the Koch snowflake, you would continue doing this pattern forever!

# Iteration

*Lesson 6 of 6, work individually or in pairs*

*Materials Needed: Pencil  
Math Journal or Notebook*

In this unit, you have done several kinds of iteration.

- Repeating a process with counting numbers to find hailstone numbers
- Repeating a process of removing center triangles to create the Sierpinski triangle
- Repeating a process of adding tents to triangle sides to create the Koch snowflake

1. **Write** How were the three processes the same? How were they different? Write a sentence or two about what was the same and what was different with the hailstone numbers, the Sierpinski triangle, and the Koch snowflake.

2. **Create** Create a new object by iteration. You can use a number rule to make something like the hailstone numbers, or you can use a picture rule to create something like the Sierpinski triangle or the Koch snowflake. Write down the rule or pattern that you use to make your object. Give it a name. Describe it to a classmate.

*Standards: Communication, patterns*