

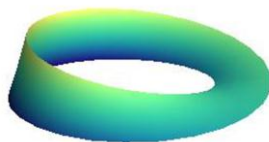
## Mobius Band **ANSWERS**

*Lesson 1 of 4, work individually or in pairs*

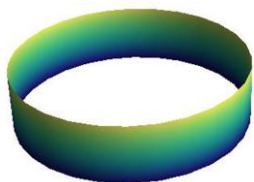
Cut a strip of adding machine tape about 2 feet long. If you don't have adding machine tape, tape together two sheets of paper along the short edge (to create an extra-tall sheet of paper that is 8.5 x 22 inches) and cut a strip 2 or 3 inches wide.

Now tape together the two ends of the strip of paper. **WAIT!** Don't tape them together so that the paper is a very short cylinder. Instead, give the strip of paper a twist by flipping one end over. Then connect the ends.

Do this



Not this



You have just created a Möbius band, a surface with some very interesting properties. It is named after August Ferdinand Möbius (1790-1868).

*Materials Needed: Pencil, Scissors, Tape  
Math Journal or Notebook  
Adding Machine Paper (or plain paper)*

1. Try to draw a line in the center of one side of your Möbius band, as shown in the picture below. What do you discover? In your math journal, write what happens.

*The line ends up going around both sides of the band!*

2. **Think About It** When you tried to draw a line in the center of one side, you may have noticed that there is a line on both sides. That means that the Möbius band has only one side. Up close, if you look at just a little tiny bit of the Möbius band, it seems to have two sides. However, if you look at the entire Möbius band, it has only one side.

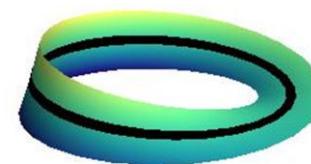
3. What do you think will happen if you trace along one edge? Write down a prediction.

*Predictions will vary.*

4. **Check** Now trace along one edge of the Möbius band. How many edges does it have?

*When you trace one edge, you end up tracing all of the edge of the band—so the band has only 1 edge.*

5. **Predict** What do you think will happen when you cut the Möbius band in half the long way?



6. **Check** Now cut the Möbius band the long way. What happened? Were you surprised? Write down what happened and whether it matched your prediction.

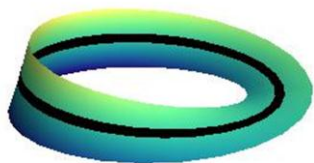
*It gets cut into one long twisted loop.*

*Standards: Geometry, prediction*

## Mobius Band **ANSWERS**

*Lesson 2 of 4, work individually or in pairs*

Make a new Mobius band, and cut it in half the long way again. (Or, use the cut-in-half Mobius band that you made on Lesson 1.)



Now you have a cut-in-half Mobius band.

1. How many twists does your cut-in-half Mobius band have?

*It has 2 twists.*

2. How many sides does your cut-in-half Mobius band have? (You can figure this out by drawing a line down the center of it, just like you did in Lesson 1.) How many edges does it have? Write down your results.

*It has 2 edges and 2 sides.*

3. **Predict** What will happen when you cut your cut-in-half Mobius band in half again? Write in your math journal what you think will happen.

*Predictions will vary.*

4. **Check** Now cut your cut-in-half Mobius band in half. Were you surprised? Write down what happened to your cut-in-half Mobius band.

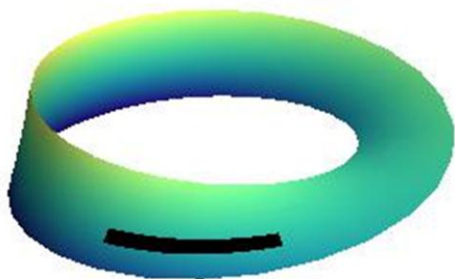
*You get two pieces that are tangled together.*

## Mobius Band *ANSWERS*

*Lesson 3 of 4, work individually or in pairs*

Make a new Mobius band. Don't cut or color it yet!

1. Think carefully about what will happen if you cut the Mobius band in thirds. That is, you will start cutting it about one-third of the way in from one edge, as shown below. Write what you think will happen when you make this cut.



*Predictions will vary.*

*Materials Needed: Pencil, Scissors, Tape  
Math Journal or Notebook  
Adding Machine Paper (or plain paper)*

2. Cut your Mobius band in thirds. This means you will start cutting one-third of the way in from one edge, and keep cutting that same distance from the edge until you get back to your starting point. Write down what happened. Were you surprised by the result?

*You get two pieces. One is longer than the other, and they are tangled together.*

3. Now what happens if you cut one of those pieces in half the long way? Pick one of the pieces to cut in half, and have one of your friends pick a different piece to cut in half. Write your prediction of what will happen, then cut, and then write down what happened.

*If you cut the short one, you get the cut-in-half Mobius band tangled around the other piece. If you cut the long one, you get 2 entangled pieces, like problem 4 in Lesson 2, that are also tangled with the short piece.*

*Standards: Geometry, prediction*

# Mobius Band **ANSWERS**

*Lesson 4 of 4, work in pairs or a small group*

Today, instead of making a Mobius band, do something different. Start with a strip of paper just like you did on Lesson 1, 2, and 3. Now, instead of making 1 twist in the piece of paper, make 2 or 3 or 4 twists before connecting the ends. Work with at least one other student, and make sure everybody makes a different number of twists in the paper.

1. In your math journal, record different experiments and their results. Make sure you keep track of how many twists your band had to start.

Number of twists: \_\_\_\_\_

Number of sides: \_\_\_\_\_

Number of edges: \_\_\_\_\_

What happens when it's cut in half: \_\_\_\_\_

What happens when the halves are cut in half:  
\_\_\_\_\_

What happens if it's cut in thirds (you'll have to make a new band to try this): \_\_\_\_\_

*Materials Needed: Pencil, Scissors, Tape  
Math Journal or Notebook  
Adding Machine Paper (or plain paper)*

## *Answers to #1:*

<i>Twists</i>	<i>sides</i>	<i>edges</i>	<i>cut in half</i>	<i>halves cut in half</i>	<i>cut in thirds</i>
<i>2</i>	<i>2</i>	<i>2</i>	<i>2 tangled</i>	<i>each piece becomes two, but is tangled with its duplicate</i>	<i>same as cut in half but skinny &amp; wide pieces</i>
<i>3</i>	<i>1</i>	<i>1</i>	<i>1 piece tangled with itself</i>	<i>2 pieces tangled together</i>	<i>2 pieces, one is knotted and tangled with the other loop</i>
<i>4</i>	<i>2</i>	<i>2</i>	<i>2 pieces, tangled in a more complicated way</i>	<i>each piece cuts in half, still tangled with the other</i>	<i>same as 2-twisted</i>

2. **Experiment** Try some other experiments. Write down what you tried and what happened.

*Look for odd/even patterns. See teacher's guide.*

3. **Compare** How were your results the same as what happened with the Mobius band? How were they different?

4. Did you find any patterns to the results of your experiments? What are those patterns? For example, if the number of twists is even or odd, does that change how many sides the Mobius band has?

*Standards: Geometry, prediction, patterns*