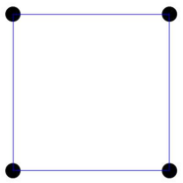


# Pick's Theorem **ANSWERS**

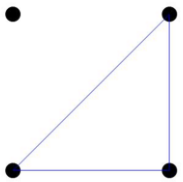
Lesson 1 of 4, work individually or in pairs

In this lesson, you will find areas of triangles on a geoboard. **If you are already familiar with the area of a triangle, you can move on to Lesson 2.**

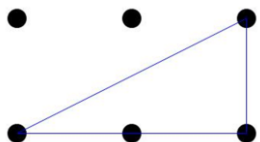
For all of the lessons in this unit, assume that the pegs on your geoboard are 1 unit apart, so the square that looks like this has an area of 1 square unit.



The triangle below would have an area of  $\frac{1}{2}$  of a square unit.

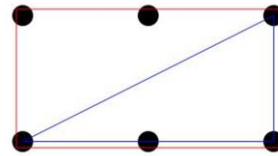


What about this triangle?



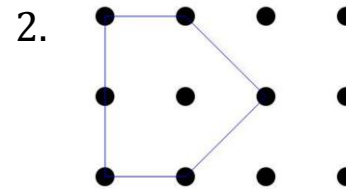
*Materials Needed: Pencil  
Math Journal or Notebook  
Geoboard or geoboard applet*

One way to think about its area is to enclose it in a rectangle.

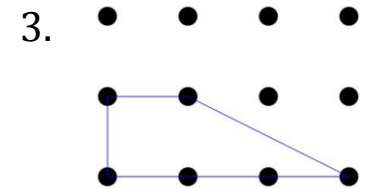


1. The area of the red rectangle is 2. That means that the area of the triangle that is half of that rectangle is 1.

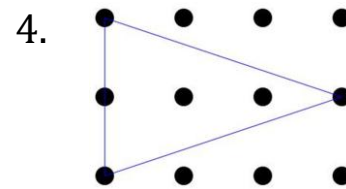
For problems 2-4, make the figures on your geoboard. Then enclose the triangles in rectangles to help you figure out the areas.



*Area = 3*



*Area = 2*



*Area = 3*

5. Check your answers with a classmate and make sure you agree!

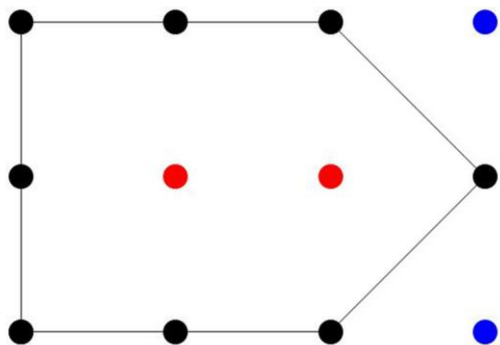
*Standards: Geometry, area*

## Pick's Theorem **ANSWERS**

*Lesson 2 of 4, work individually or in pairs*

A polygon that you can make on a geoboard is called a *lattice polygon*. In this activity you will learn a new way to find the area of a lattice polygon.

1. Make a polygon that looks like this.



If we assume that the pins are 1 unit apart, what is the area of the polygon you made?

**Area = 5**

This polygon has 8 boundary pins (colored black) and 2 interior pins (colored red). If we call the number of interior pins  $I$  and the number of boundary pins  $B$ , we can say  $B = 8$  and  $I = 2$ .

2. Make two different polygons with  $B = 6$  (6 boundary pins) and  $I = 0$  (0 interior pins). Draw your polygons and figure out the area of both of them.

*Materials Needed: Pencil  
Math Journal or Notebook  
Geoboard or geoboard applet*

3. Now change one of your polygons to have  $B = 7$  and  $I = 0$ . Draw the polygon in your math journal, write down that  $B = 7$ , and find the area of the polygon.

**area =  $2 \frac{1}{2}$**

4. Make a table that describes the number of boundary pins, number of interior pins, and the area. Draw all of your polygons in your math journal.

B (boundary pins)	I (interior pins)	Area
6	0	<b>2</b>
7	0	<b><math>2 \frac{1}{2}</math></b>
8	0	<b>3</b>
9	0	<b><math>3 \frac{1}{2}</math></b>

5. **Predict** What is the area of a polygon with  $B = 12$  and  $I = 0$ ?

**area = 5**

6. **Check** Make a polygon with  $B = 12$  and  $I = 0$  and see if your prediction is true.

7. **Extend** Do you see a pattern in what happens to the area as the number of boundary pins increases? What do you think the rule is for the area of a polygon with  $I = 0$ ?

**As you increase the number of boundary pins by 1, you increase the area by  $\frac{1}{2}$ . (Some students may discover that the area is  $\frac{B}{2} - 1$ .)**

*Standards: Geometry, area, patterns*

## Pick's Theorem *ANSWERS*

*Lesson 3 of 4, work individually or in pairs*

In the last lesson, you had a formula for the area of a polygon with  $I = 0$ . Today you will start to figure out what happens when  $I \neq 0$ .

1. Draw this table in your notebook, and fill in the area column. For each row in the table, draw in your math journal the polygon you used. Label each polygon with the correct numbers for B, I, and area.

B (boundary pins)	I (interior pins)	Area
6	0	2
6	1	3
6	2	4
6	3	5
7	0	$2 \frac{1}{2}$
7	1	$3 \frac{1}{2}$
7	2	$4 \frac{1}{2}$
8	0	3
8	2	5
8	4	7

2. For each line of the table, did it matter what shape the polygon was, or did just the numbers B and I matter?

*Just the numbers B and I matter.*

*Materials Needed: Pencil  
Math Journal or Notebook  
Geoboard or geoboard applet*

## Pick's Theorem **ANSWERS**

*Lesson 4 of 4, work individually or in pairs*

Look at the table you made in Lesson 3 to figure out the answers to the questions in this lesson. (If most of the rows in the table make a particular pattern, but one row doesn't, look again at that row to see if you made a mistake in the number of pins or in the area.)

1. A polygon with  $B = 10$  and  $I = 3$  has area 7. What do you think will be the area if  $B = 10$  and  $I = 4$ ?

*Area = 11*

2. **Predict** As you add interior pins to a lattice polygon, how does the area change? Write down a rule about the number of interior pins and the area of a polygon.

*Increasing  $I$  by 1 increases area by 1.*

*(Some students may put this together with the rule about  $B$  and see that the area of the polygon is*

*$\frac{B}{2} + I - 1$ .)*

*Materials Needed: Pencil  
Math Journal or Notebook*

3. **Check** Make a new polygon that you haven't made already. Check to see if your rule works with this new polygon.

4. **Share** Share this rule with a classmate, and see if your classmate found the same rule. Were you surprised by anything about the pattern you discovered?

The rule you just discovered is called Pick's Theorem, named after Georg Alexander Pick. (A good biography of Pick can be found at <http://www-history.mcs.st-and.ac.uk/Biographies/Pick.html>.)

*Standards: Geometry, patterns*