

Teacher Guide for Pick's Theorem

Special notes and timing

In a class of 4th graders, the last three lessons was about an hour block. The longest lesson is Lesson 3, because there are so many polygons for the students to make and think about. The first lesson is optional—if students know the formula for an area of a triangle, they can skip this lesson. If they don't know the formula, they should do lesson 1. The formula for the area of a triangle is not necessary to complete these lessons.

Learning Objectives

Students will:

- Describe the relationship between boundary pins, interior pins, and area of a lattice polygon.
- Recognize that for lattice polygons, the number of boundary pins and interior pins completely determines the area of the polygon—the shape doesn't matter. (This is surprising!)

Materials

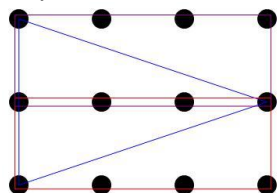
- Math journal or notebook
- Geoboard or geoboard applet (there are working applets at <http://www.cut-the-knot.org/Curriculum/Geometry/Geoboard.shtml> and at <http://www.nctm.org/standards/content.aspx?id=25008>).

Mathematical Notes

The most general result of Pick's theorem is that for a lattice polygon, the area A of a polygon with B boundary pins and I interior pins is given by $A = \frac{B}{2} + I - 1$. Students will probably not come up with the formula, but should recognize the following relationships:

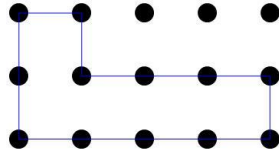
- If you leave the number of interior pins the same but increase the number of boundary pins by 1, the area goes up by $\frac{1}{2}$. (Lesson 2)
- If you leave the number of boundary pins the same but increase the number of interior pins by 1, the area goes up by 1. (Lessons 3-4)
- The shape of the polygon doesn't matter—just the number of interior and boundary pins. (This is pretty surprising!)

In Lesson 1, students may be particularly troubled by problem 4. One way to do that problem is to make two additional rectangles, as shown below. A common error for this triangle's area is for students to think that the triangle is about one-third of the large enclosing rectangle. Help students see it another way.



In Lesson 2, the students are asked to create a polygon with 12 boundary pins and 0 interior pins. If the geoboard is 5 pins long, the students can't just use a rectangle on the geoboard to make this polygon. Possible solutions include imagining the polygon and drawing it in their notebook, placing two

geoboards next to each other to create a 6-pin-long rectangle, or making an L-shaped polygon on the geoboard.



Helping Questions

How might you begin?

What pattern do you see?

If you add just one more boundary pin without adding more interior pins, what happens to the area?

If you add just one more interior pin without adding more boundary pins, what happens to the area?

Compare your results with a different polygon. Do you notice any relationships?

What do you notice? What else do you notice?

Have you asked one of your classmates if he or she could see a pattern?

Why do you think that happens? (*Students may not know the answer to this question, but it helps them to think about it and try to explain it.*)

What did you try that didn't work? Can you learn something from that?

Can you explain it in a different way?

Assessment Options

- Look at the students' math journals. Ensure that they write down enough details so that they could look back and understand their work without having the lesson cards available. They should have drawn pictures of all the polygons they used to fill in the tables in lessons 1 and 2.
- Students should be able to state two rules about Pick's theorem. In the Lesson 2, they should recognize, "as B goes up by 1, the area goes up by $\frac{1}{2}$." In Lessons 3-4, they should recognize, "as I goes up by 1, the area goes up by 1."
- Have students find the areas of new lattice polygons using Pick's Theorem.

Extensions

Lesson 4 has a link to a biography of Pick. Students could read about him and write a report on his career.

Make sure that students understand that Pick's theorem applies only to lattice polygons—ones that can be made on a geoboard. For example, there is no way to find the area of an equilateral triangle using Pick's Theorem, because an equilateral triangle is not a lattice polygon.

Teacher Reflection

- Which students were able to see the patterns quickly, and which students struggled to see the patterns?
- Did some students get frustrated if they couldn't find a pattern immediately? What can you do to ease that frustration?
- Did some students work better individually or in pairs/small groups? What support can you provide to the students to work outside their comfort zone?
- What were the greatest challenges for the students?

Standards Addressed

Common Core State Standards (and Colorado Academic Standards in Mathematics)

- 2. Patterns, Functions, and Algebraic Structures
- 4. Shape, Dimension, and Geometric Relationships

NCTM (National Council of Teachers of Mathematics) Content Standards

Algebra
Geometry

NCTM Process Standards

Problem Solving
Reasoning and Proof
Connections
Representation

References Used

Investigating Pick's Theorem, *NCTM Illuminations website*,
<http://illuminations.nctm.org/LessonDetail.aspx?id=U167>, accessed February 13, 2012.

Bogolmony, Alex. Pick's Theorem: An Interactive Column Using Java Applets, *Cut the Knot! website*,
<http://www.cut-the-knot.org/ctk/Pick.shtml>, accessed February 13, 2012.

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