

## Games of Skill **ANSWERS**

Lesson 1 of 9, work in pairs

Materials Needed: Pencil  
Math Journal or Notebook

### 21 (basic version)

The goal of the game is to get the other player to say the number 21. The person who says 21 loses.

The first person starts by saying 1.

At each turn, the player can count 1 or 2 times.

Here's an example of a game between Alice and Bob:

Alice: 1	Bob: 2
Alice: 3, 4	Bob: 5, 6
Alice: 7	Bob: 8
Alice: 9, 10	Bob: 11
Alice: 12, 13	Bob: 14
Alice: 15	Bob: 16, 17
Alice: 18	Bob: 19, 20
Alice: 21	

Alice loses because she said 21.

1. Play this game several times with a classmate. Take turns going first. Record all of the numbers you said in the game, and who won each game.

2. What numbers are winning numbers, assuming that both players are playing with a good strategy? **For example, 20 is a winning number because the opponent is required to say 21.** What other numbers can you say that will guarantee a win?

*20, 17, 14, 11, etc (numbers that are one less than multiples of 3). Students will probably get the largest few numbers.*

3. Assume both players are playing with the best strategy. For example, in the game shown, Bob played the right strategy once Alice said "18". If Bob had said "19", then Alice could have won. But, since Bob played with the right strategy, he won once Alice said 18, so **18 is a losing number**. What numbers are guaranteed losing numbers?

*21, 19, 18, 16, 15, 13, 12, etc (any number that is not a winning number). Students will probably get the largest few numbers.*

4. **On Your Own** Teach this game to a family member. See how quickly they learn the right strategy. Report back on it tomorrow.

Standards: Reasoning, planning

## Games of Skill **ANSWERS**

*Lesson 2 of 9, work in pairs*

### **21 (harder version)**

Repeat the game from Lesson 1, but now you can say 1, 2, or 3 numbers.

1. Play this game several times with a classmate. Take turns going first. Record all of the numbers you said in the game, and who won each game.

2. What numbers are winning numbers, assuming that both players are playing with a good strategy? **For example, 20 is a winning number because the opponent is required to say 21.** What other numbers can you say that will guarantee a win?

*20, 16, 12, 8, 4 (multiples of 4); students will probably get the largest couple of these*

*Materials Needed: Pencil  
Math Journal or Notebook*

3. Again, assume both players are playing with the best strategy. In this game, **17 is a losing number** if both players play with the right strategy. What numbers are guaranteed losing numbers?

*21, 19, 18, 17, 15, 14, 13 (all but the winning numbers); students will probably get the largest several of these*

4. **On Your Own** Teach this game to a family member. See how quickly they learn the right strategy. Report back on it tomorrow.

*Standards: Reasoning, planning*

## Games of Skill **ANSWERS**

Lesson 3 of 9, work in pairs

### Keep-Away

Draw a 5-square row, as shown.



In each turn, the player can put an X in any square she chooses, **except a square next to another X**. The players take turns playing, and the last player to put an X in the board wins.

Example game:

Blue goes first, and plays



Red cannot play in the shaded squares, so plays in another square.



Red wins because blue does not have another place to play.

*Materials Needed: Pencil  
Math Journal or Notebook  
Colored Pens or Pencils*

1. Play this game several times in your math journal. Keep track of who wins—player 1 or player 2. It may be easier to keep track of if you use different colors to write. Take turns being the first player. Play until you think you know how to beat your opponent.

2. Does it matter who goes first? Does one player always win if both players play with the best strategy?

*Player 1 can always win by putting the X in the middle square.*

3. **Explain** Explain in your math journal what you think the winning strategy is.

*Standards: Reasoning, planning*

## Games of Skill **ANSWERS**

Lesson 4 of 9, work in pairs

### **Keep-Away** (harder versions)

1. Play the game from Lesson 3 again, this time using 6 squares. Play in your math journal, and keep track of whether the first or second player wins. Keep playing until you think you know the winning strategy. Write this winning strategy in your math journal.



*Materials Needed: Pencil  
Math Journal or Notebook  
Colored Pens or Pencils*

2. Play again with 7 or 8 squares. Could you guess the winning strategy right away? Again, track your games in your math journal.

*7 squares: The first player can always win by playing in the middle square.*

*8 squares: Player 2 can win! If player 1 plays in the 4<sup>th</sup> box, player 2 should play in the 6<sup>th</sup>. If player 1 plays in the 3<sup>rd</sup> box, player 2 should play in the 7<sup>th</sup>. If player 1 plays in the 2<sup>nd</sup> box, player 2 should play in the 6<sup>th</sup> box. If player 1 plays in the first box, player 2 should play in the 5<sup>th</sup> box.*

***Not all students will have full strategy—teacher should look for some good reasoning.***

3. Now play with 9 squares. This time the strategy is harder! Can you figure it out? Track your games in your math journal to help you figure out the strategy, and then write down the winning strategy.

*Player 1 should play in the middle box. From here, player 2 will play somewhere, and then player 1 should mirror what player 2 did (for example, if player 2 plays in the box that is 2<sup>nd</sup> from the left, then player 1 should play in the box that is 2<sup>nd</sup> from the right.)*

***Not all students will have full strategy—teacher should look for some good reasoning.***

*Standards: Reasoning, planning*

# Games of Skill *ANSWERS*

Lesson 5 of 9, work in pairs

*Materials Needed: Pencil  
Math Journal or Notebook*

## Triplets

Make a 3x3 grid in your math notebook. The players take turns placing the same kinds of marks in the boxes. The first player to finish a 3-in-a-row wins. The 3-in-a-row can be horizontal, vertical, or diagonal. See the example game below.

	X	

Player 1

	X	
X		

Player 2

	X	
X		
		X

Player 1

X	X	
X		
		X

Player 2

X	X	X
X		
		X

Player 1

Player 1  
Wins!

1. Play the game several times with a classmate. Take turns being the first player. Is the first player or the second player more likely to win?

*Player 1 seems to win more often.*

2. Have you figured out how to win the game? Write your winning strategy in your math journal.

*Player 1 can win the fastest by playing in the middle box.*

3. Now use that strategy to play the same game on a 4x4 grid. Now the strategy is more complicated!

*Standards: Reasoning, planning*

# Games of Skill **ANSWERS**

Lesson 6 of 9, work in pairs

## **Triplets** (continued)

Continue with your experiments from Triplets from the Lesson 5.

1. How many markers can be placed on a 4x4 board before the next player has no safe move? For example, if you have the arrangement shown here, there are no safe places for the next player to move, because any move leads to a 3-in-row on the following turn.

	X		
X			
		X	

Can you have 4 markers on the board before there is no safe move? What about 5? Draw the diagram in your math journal that shows this number of markers. How do you know that you can't have more markers, perhaps in a different arrangement?

		X	
X			
			X
	X		

*This shows an arrangement with 4 markers.  
There is none with 5 markers.*

*Materials Needed: Pencil  
Math Journal or Notebook*

2. What do you think the winning strategy is for the game with the 4x4 board? How does your strategy change if you are the first player or the second player?

*Player 1 can win by playing in any of the squares highlighted below. Player 2 has to hope player 1 makes a mistake.*


3. When you think you understand the 4x4 game, try a 5x5 grid. Answer some of the same questions that you asked for the 3x3 and 4x4 games. Record your results in your math journal.

*Player 1 can win when playing in any of the squares highlighted below.*


*Standards: Reasoning, planning*

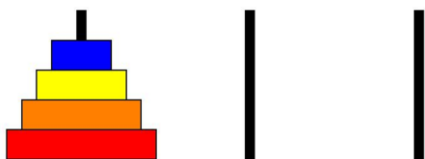
## Games of Skill **ANSWERS**

Lesson 7 of 9, work individually or in pairs

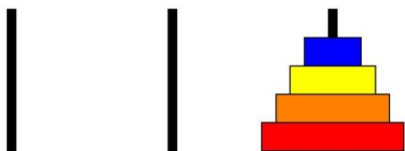
### *Towers of Hanoi puzzle*

This game is a one-player puzzle, but you can work cooperatively with a classmate to figure it out. In the puzzle you always have 3 pegs. You have a tower of different sized disks, and need to move them from the left-most peg to the right-most peg.

Start like this:



End like this:

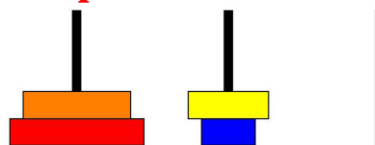


Rules of movement:

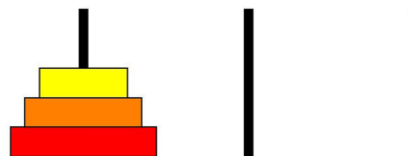
- You can move only one disk at a time.
- You can only move the top disk from a stack.
- You can move the disk to any peg you choose.
- You may NOT place a larger disk on top of a smaller disk.

*Materials Needed: Pencil  
Math Journal or Notebook  
Colored Pens or Pencils*

**The last rule means that the picture below is not permitted!**



In the easiest version of the puzzle, start with a stack of 3 disks.



1. Solve this puzzle. Keep track of the total number of moves that you use to solve the puzzle.
  2. Solve the problem again, and see if you can do it in fewer moves.
- The best possible is 7 moves.*
3. Once you are confident that you have found the solution with the fewest moves, try to solve the puzzle using 4 disks.

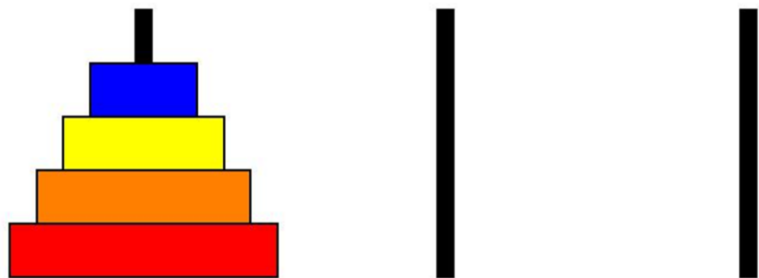
*Standards: Reasoning, planning*

## Games of Skill **ANSWERS**

Lesson 8 of 9, work individually or in pairs

Materials Needed: Pencil  
Math Journal or Notebook  
Colored Pens or Pencils

### *Towers of Hanoi* (day 2)



Review your work on the Towers of Hanoi puzzle from Lesson 7.

1. Do the puzzle again with 4 disks. Keep track of the number of moves it takes you.

*The best possible is 15 moves.*

2. Solve the puzzle again, and see if you can do it in fewer moves. Can you do it in fewer than 20 moves?

3. Once you get to a really good solution with 4 disks, try the puzzle again with 5 disks. How many moves does it take you? What is the best that you can do?

*The best possible with 5 disks is 31 moves.*

3. **Explain** Explain in your math journal how you can solve this puzzle in the fewest number of moves. Try to explain it in general terms instead of step by step.

*One way to explain: Move all of the smaller circles off of the largest one, and get them stacked on the center peg. (For 4 disks, you do this by first stacking the smallest 2 circles on the right peg, then moving the 3<sup>rd</sup> largest to the center, then restack the smallest two circles on the center.) Then move the large circle to the right peg, and move the stack of smaller circles by mirroring all of the previous moves.*

*The sophistication of explanations will vary widely—this is a pretty hard task.*

Standards: Reasoning, planning, communication



## Games of Skill

*Lesson 9 of 9, work in pairs*

*Materials Needed: Pencil  
Math Journal or Notebook*

In this unit, you have learned several strategy and thinking games.

1. **Create** Create your own variation of one of these games. Give it a name. Write down the rules in your math journal.

2. Teach your new game to a friend. Play it, and then analyze the strategy in the same way that you have in other lessons in this unit.

3. **Explain** Write in your journal the answers to the following questions. Is there a clear winning strategy? Do you want to be the first player or second player? Describe the winning strategy and how you know it works.

*Standards: Reasoning, planning, communication*