Teacher Guide for Graph Theory

Learning Objectives

Students will:

- Define a mathematical graph, identifying edges and vertices.
- Represent real-life situations with mathematical graphs.
- Determine which graphs have Euler circuits, and describe why that is an interesting question.
- Recognize patterns that arise in various graph problems.
- Determine the shortest path in traveling salesman problems.

Materials

- Math journal or notebook
- Highlighter for lesson 7

Helping Questions

How might you begin?
What does that word mean?
What are you trying to figure out?
What pattern do you see?
What is the next step (in the process, in the pattern)? Does that help you see any patterns?
Compare your results with XX. Do you notice any relationships?
What do you notice? What else do you notice?
Have you asked one of your classmates if he or she could see a pattern?
Why do you think that happens? (*Students may not know the answer to this question, but it helps them to think about it and try to explain it.*)
What did you try that didn't work? Can you learn something from that?
Can you explain it in a different way?

Assessment Options

- Look at the students' math journals. Ensure that they write down enough details so that they could look back and understand their work without having the lesson cards available.

Mathematical Notes

The number of computations involved in the traveling salesman problem depends on the number of vertices in the graph. For a 4-vertex graph, there are $3 \times 2 = 6$ different paths, and only 3 paths if forward and backward paths are considered to be the same. For a 5-vertex graph, there are $4 \times 3 \times 2 = 24$ paths (12 if forward and backward are equivalent). For a 6-vertex graph, there are $5 \times 4 \times 3 \times 2 = 120$ paths.

You can go to http://www.tsp.gatech.edu/optimal/index.html to see some of the current records for a solution of the Traveling Salesman Problem. This page includes a lot of information about programming, which you probably don't want the students to learn, but helps them see that the challenges of solving this problem are not just something that you can do with a simple computer program.

The number of edges in a complete graph (Lesson 7) is a triangular number, as discussed in the *Polygonal Numbers* unit.

Extensions

- Have the students consider how a GPS unit in the car or phone evaluates the quickest path. It is related to the traveling salesman problem of Lesson 5. Have students go to a map website and look at a map of their neighborhood and different paths to get to school.
- To extend Lesson 5, have students develop their own vacation itinerary. They need to pick a starting location and several other vacation locations. They can use a web-based mapping site (such as Google Maps or MapQuest) to find the distances between the locations and solve the traveling salesman problem.
- If you want, have students use factorial notation to describe the number of paths in a traveling salesman problem.
- Have students develop their own report on the topics in the lesson. They could do a slide show, poster, verbal presentation, or written report.

Teacher Reflection

- How did the students react to a new definition of the familiar word *graph*?
- Did students recognize different ways to represent the same graph? How could you help them see the similarities or differences?
- Which students were able to see the patterns quickly, and which students struggled to see the patterns?
- Did some students get frustrated if they couldn't find a pattern immediately? What can you do to ease that frustration?
- Did some students work better individually or in pairs/small groups? What support can you provide to the students to work outside their comfort zone?
- What were the greatest challenges for the students?

Standards Addressed

<u>Common Core State Standards (and Colorado Academic Standards in Mathematics)</u>

1. Number Sense, Properties, and Operations
2. Patterns, Functions, and Algebraic Structures

<u>NCTM (National Council of Teachers of Mathematics) Content Standards</u>

Number and Operations
Algebra
Geometry

<u>NCTM Process Standards</u>

Problem Solving
Reasoning and Proof
Communication
Connections
Representation

References Used

Charles D. Miller, Vern E. Heeren, and John Horsby, *Mathematical ideas*, 9th edition. Addison Wesley, Boston, 2001.

Georgia Tech's Traveling Salesman Problem website, http://www.tsp.gatech.edu/, accessed February 28, 2012.

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