Teacher Guide for Iteration

Learning Objectives

Students will:

- Describe step-by-step processes.
- Discover patterns related to iteration.
- Create their own object through iteration.

Materials

- Math journal or notebook
- Scissors and tape
- Triangle graph paper (see accompanying master)

Helping Questions

How might you begin?
What pattern do you see?
What is the next step (in the process, in the pattern)? Does that help you see any patterns?
Compare your results with XX. Do you notice any relationships?
What do you notice? What else do you notice?
Have you asked one of your classmates if he or she could see a pattern?
Why do you think that happens? (*Students may not know the answer to this question, but it helps them to think about it and try to explain it.*)
What did you try that didn't work? Can you learn something from that?
Can you explain it in a different way?

Assessment Options

- Look at the students' math journals. Ensure that they write down enough details so that they could look back and understand their work without having the lesson cards available.
- Students should be able to explain that iteration is where the answer from one step is the starting point for the next step.

Mathematical Notes

For Lesson 4, here is an explanation of the filled-in table from question 1-3:

Step	Number of new white triangles	Total number of white triangles	Total number of blue triangles	Total number of triangles (white + column 4)
1	1	1	3	4
2	3	4	9	13
3	9	13	27	40
4	27	40	81	121
5	81	121	81*3=243	121+243=364
6	243	364	3*243=729	729+364=1093

The previous column 4 and 5 entries become the new column 2 and 3 entries. This is because the new white triangles come from the old blue triangles. The total number of white triangles is the old number of triangles, because all of the blue triangles contributed a new white triangle and the old white triangles need to get counted again. Another way to see some of these numbers is that the number of blue triangles is multiplied by 3 each time, and the number of white triangles is the previous number of white triangles, times 3, then with 1 added ($13 \times 3 + 1 = 40$ for example).

For lesson 6, you may need to be more directive with students than the lesson cards are. You could ask questions such as:

- What other number rules could you use? (Students will have to work hard to figure out how to make sure their answers are counting numbers.)
- What if you used squares to build a side instead of triangles?
- What could you do if you started with a square instead of a triangle?
- What if you used a really large or really small triangle to modify the Koch triangle?
- Can you look online to see what other ideas you can find?

If your students are working with S/T Math (Jiji), the challenge game "Big Seed" is based on iteration—the answer to one step is the starting point for the next step. In iteration, the very first starting point (such as the beginning number in hailstones or the initial triangle for the Sierpinski triangle) is called the "seed."

Extensions

Lesson 6 has one extension plan. Encourage students to be creative. Perhaps they can use color to make their creation visually appealing.

Students could research self-similarity and fractals. The website http://math.rice.edu/~lanius/fractals/self.html describes self-similarity. There is a good website, http://classes.yale.edu/fractals/Panorama/welcome.html, that has a lot of examples of fractals (self-similar objects). The list down the left side of the page is great for student exploration. It certainly doesn't give complete information on any of the topics, but helps the students see the wealth of topics available for study.

Teacher Reflection

- Which students were able to see the patterns quickly, and which students struggled to see the patterns?
- Which students were most interested in the number iteration and which were more interested in geometric iteration? What content can you find to challenge these students further?
- Did some students get frustrated if they couldn't find a pattern immediately? What can you do to ease that frustration?
- Did some students work better individually or in pairs/small groups? What support can you provide to the students to work outside their comfort zone?
- Did the students enjoy having to create their own iteration process, or did the idea of creativity in mathematics seem new and challenging?
- What were the greatest challenges for the students?

Standards Addressed

Common Core State Standards (and Colorado Academic Standards in Mathematics)

1. Number Sense, Properties, and Operations
2. Patterns, Functions, and Algebraic Structures
4. Shape, Dimension, and Geometric Relationship

NCTM (National Council of Teachers of Mathematics) Content Standards

Number and Operations
Algebra
Geometry
Measurement

NCTM Process Standards

Problem Solving
Reasoning and Proof
Communication
Connections
Representation

References Used

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