

## Teacher Guide for Mobius Band

### Special notes and timing

This lesson has been successfully tested with talented 2<sup>nd</sup>-graders.

The time for students to complete all 4 lessons is 60-90 minutes.

This lesson is easily adaptable for a large-group activity instead of having students working at their own pace in small groups. If you use this for a large-group activity, the “Mobius band worksheet” is a good mechanism for the students to record their thoughts as they explore the Mobius bands.

### Learning Objectives

Students will:

- Create a Mobius band
- Recognize that a Mobius band has only one side and one edge
- Predict what happens when a Mobius band is cut

### Materials

- Math journal or notebook
- Adding machine paper or plain printer paper
- Scissors and tape

### Helping Questions

Were you surprised by what happened?

What do you notice? What else do you notice?

Have you asked one of your classmates if he or she discovered the same thing?

Why do you think that happens? (*Students may not know the answer to this question, but it helps them to think about it and try to explain it.*)

What did you try that didn't work? Can you learn something from that?

Can you explain it in a different way?

### Assessment Options

- Look at the students' math journals. Ensure that they write down enough details so that they could look back and understand their work without having the lesson cards available.
- Have students report important qualities about Mobius Bands.
- Have students write on a notecard two thoughts and one question that arose from their activities.

### Mathematical Notes

The teacher needs to understand what a Mobius band looks like to check to make sure students make one correctly at the start of lesson 1. Once students make one Mobius band, they understand the idea and can make future ones.

Students may be surprised by the fact that the Mobius Band has only one side, as explained in problem 2 of Lesson 1. It also has only one edge. The reason that it is still in only one loop after it is cut in half is

that it used to have only one edge (one edge of the new loop), and by cutting you have added a 2<sup>nd</sup> edge to the Mobius bands.

There is a very good explanation of why the Mobius Band behaves the way it does in the YouTube video <http://www.youtube.com/watch?v=Am-a5x9DGjg> (entitled Math Improv: Fruit by the Foot). It is appropriate for students to watch after Lesson 3.

Students are most surprised by the results of the experiment about cutting the Mobius band in thirds. Here's one explanation: The original Mobius band had one edge. You cut that edge off by cutting the band in thirds (that's where the long piece comes from). Then when you cut the edge off, the middle was still left (that's where the other shorter piece comes from).

An even number of twists creates a 2-sided, 2-edged object that, when cut in half, ends up in 2 pieces. (And cutting a third off of this even-twisted object really acts the same as cutting it in half, except one of the pieces ends up wider than the other.) An odd number of twists creates a 1-sided, 1-edged object that, when cut in half, is still one piece. If there are 3 or more twists, the object cut in half will be knotted with itself.

Students are interested in the fact that the new loops made from Mobius bands are almost as tall as they are. This can lead to a nice discussion about the length of the new object being twice the length of the starting object.

### **Extensions**

Have students watch the YouTube video <http://www.youtube.com/watch?v=4mdEsoulXGM> (Mobius story: Wind and Mr. Ug) and predict what will happen with Wind and her dog.

See if students can find a pattern relating the number of twists to the number of sides or edges of a generalized Mobius Band.

Students could study the life of August Ferdinand Mobius and write a report on him and his mathematics. A good starting point is the MacTutor History of Mathematics biography at <http://www-history.mcs.st-and.ac.uk/> (search on Mobius). Students will learn that Mobius did not really invent the Mobius band, but instead Johann Benedict Listing did. Ask the students if they think that it's fair that we call it a Mobius band instead of a Listing band!

### **Teacher Reflection**

- How did students react to their predictions being right or wrong?
- If students became frustrated with incorrect predictions, what can you do to ease that frustration?
- What were the greatest challenges for the students?
- What did the students learn in their experiments that surprised them (or you)?

### **Standards Addressed**

Common Core State Standards (and Colorado Academic Standards in Mathematics)

2. Patterns, Functions, and Algebraic Structures

#### 4. Shape, Dimension, and Geometric Relationship

##### NCTM (National Council of Teachers of Mathematics) Content Standards

##### Geometry

##### NCTM Process Standards

Problem Solving  
Reasoning and Proof  
Communication  
Connections  
Representation

##### **References Used**

MacTutor biography of August Ferdinand Mobius, <http://www-history.mcs.st-andrews.ac.uk/Biographies/Mobius.html>, accessed March 20, 2012.

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