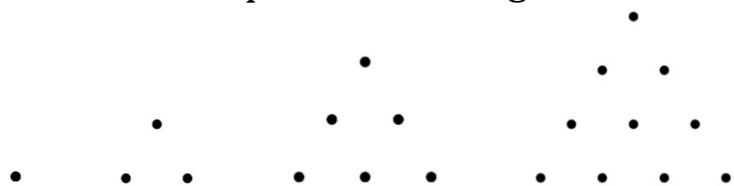


Polygonal Numbers

Lesson 1 of 6, work individually or in pairs

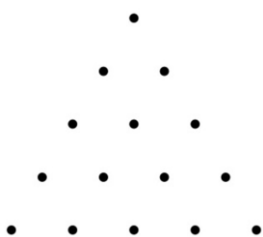
Materials Needed: Pencil
Math Journal or Notebook

Look at this sequence of triangles.



Each triangle is an equilateral triangle. (The first one doesn't look like a triangle, but it would be the triangle that you could make if every side has only one dot.) The second triangle has 2 dots on each side, the third has 3 dots on each side, and so on. If we want to count the number of dots in each triangle, the first one has 1 dot, the second has $1 + 2 = 3$ dots, the third one has $1 + 2 + 3 = 6$ dots, and the fourth one has $1 + 2 + 3 + 4 = 10$ dots. Because the numbers 1, 3, 6, and 10 come from counting dots in a triangle, they are called **triangular numbers**.

If we want to find the next triangular number, we could draw another triangle like this and count the dots



or we could just imagine that the next triangle has $1 + 2 + 3 + 4 + 5 = 15$ dots.

1. Find the next five triangular numbers. In your math journal, write the triangular numbers as a list of numbers:

1, 3, 6, 10, 15, __, __, __, __, __

Circle this list of numbers—you are going to use it in several lessons.

2. The first triangular number is 1. The 2nd triangular number is $1 + 2$. The 3rd triangular number is $1 + 2 + 3$ and the 4th is $1 + 2 + 3 + 4$. What numbers would you add to find the 20th triangular number? **(Don't add it up, just write down the addition problem.)**

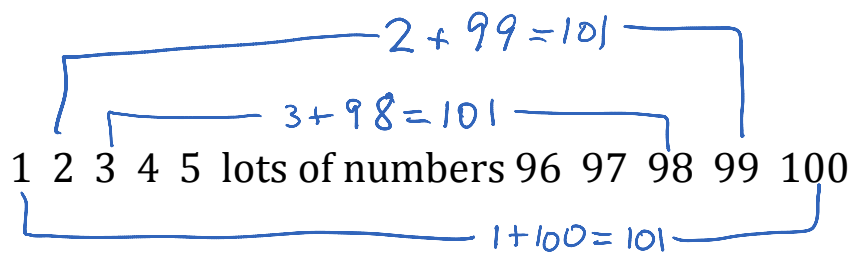
3. Look at the list of list of triangular numbers in problem 1. They make the pattern odd, odd, even, even, odd, odd, even, even, and so on. Do you think this same pattern continues? Why or why not?

Standards: Patterns, representation

Polygonal Numbers

Lesson 2 of 6, work individually or in pairs

There is a story that is told about the mathematician Johann Carl Friedrich Gauss, who lived in Germany from 1777-1855. The story says that when he was in elementary school, Gauss' teacher told him to add up all of the numbers from 1 to 100. Within less than a minute, Gauss had the answer. Nobody really knows if this story is true, but it tells something about finding triangular numbers. Here's the method that people think he used: Match up all of the numbers in pairs, so 1 matches with 100, 2 matches with 99, 3 matches with 98, and so on (see the picture below). Each pair adds up to 101. There are 50 of these pairs (if you keep going, you find that 50 and 51 match up). So the sum of all of the numbers from 1 to 100 is $101 \times 50 = 5,050$.



Materials Needed: Pencil
Math Journal or Notebook

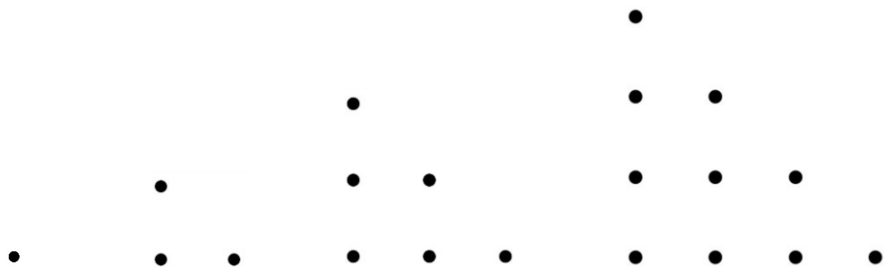
1. Look at question 2 of Lesson 1, where you wrote the sum for the 20th triangular number.
 - a. Match up the numbers the way Gauss did. What is the sum of each match?
 - b. How many matches (pairs) of numbers do you have?
 - c. Multiply your answers from parts a and b to find the value of the 20th triangular number.
2. Try to use Gauss' method to find the 7th triangular number, $1 + 2 + 3 + 4 + 5 + 6 + 7$. It doesn't quite work, but you can modify the method so that it does. *Once you find the sum, make sure it matches a number in your list from problem 1 on Lesson 1.*
3. Use this method to find the 1,000th triangular number.
4. How would you use a similar method to find the sum $2 + 4 + 6 + 8 + \dots + 198 + 200$? Be careful when you count the number of pairs of numbers. (Hint: Figure out how many numbers are added in $2 + 4 + 6 + 8 + \dots + 198 + 200$.)

Standards: Numbers, multiplication, patterns

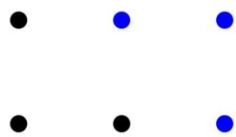
Polygonal Numbers

Lesson 3 of 6, work individually or in pairs

Another way to find triangular numbers is to think about the triangles being right triangles instead of equilateral triangles.



Now we can put together two of the same triangular numbers to make a rectangle, like this:



The rectangle with 2 of the triangular numbers had $3 \times 2 = 6$ dots, so each of the triangular numbers is $6 \div 2 = 3$ dots.

If we do the same thing with the third triangular number, we see that two of them together have $4 \times 3 = 12$ dots, so the third triangular number is $12 \div 2 = 6$.

Materials Needed: Pencil
Math Journal or Notebook

If we put it all together, we see that
the 2nd triangular number is $3 \times 2 \div 2 = 3$
the 3rd triangular number is $4 \times 3 \div 2 = 6$
the 4th triangular number is $5 \times 4 \div 2 = 10$

1. Do you see the pattern in red? Use that pattern to find the 100th triangular number. Write the answer in your notebook.
2. **Check** Compare your answer to problem 1 with the story about Gauss in Lesson 2. Did you get the same answer? If you didn't, go back and check your work.
3. **Challenge Question** Why does looking at the rectangles do the same kind of matching that Gauss did? Write one or two sentences to explain why the picture shows Gauss' matching method.

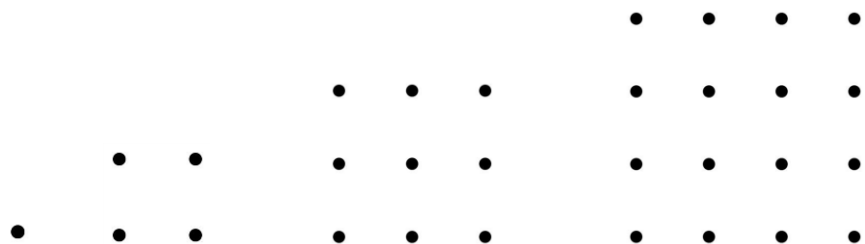
Standards: Numbers, geometry, patterns

Polygonal Numbers

Lesson 4 of 6, work individually or in pairs

Materials Needed: Pencil
Math Journal or Notebook

Now let's look at square numbers.



1. How many dots are in each square?
2. What number do you multiply by itself to get the number of dots in each square?
3. What is the next square number?

4. The first square number is $1 \times 1 = 1$. The 2nd square number is $2 \times 2 = 4$. The 3rd square number is $3 \times 3 = 9$. What is the 20th square number?

5. Now look at the differences between the square numbers. How many dots were added to the first square to get the second square? How many dots were added to the second square to get the third square? Make a table like this in your notebook:

To get from the 1 st to the 2 nd , add	3 dots
to get from the 2 nd to the 3 rd , add	5 dots
to get from the ___ to the ___, add	

6. **Communicate** Look at the table above and write the pattern that describes what happens from one row to the next row. Write a sentence or two to describe that pattern.

Standards: Numbers, geometry, patterns

Polygonal Numbers

Lesson 5 of 6, work individually or in pairs

So far we have computed *triangular numbers*

1, 3, 6, 10, 15, 21, 28, 36, 45, 55, and so on.

We also computed the *square numbers*

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, and so on.

1. Find out what happens when you add consecutive triangular numbers. (*Consecutive* means one right after the other.) List the sums neatly in your notebook, as shown. Do this for all numbers on the list, so you will have 9 different sums listed.

$$1 + 3 = 4$$

$$3 + 6 = \underline{\quad}$$

$$6 + 10 = \underline{\quad}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

*Materials Needed: Pencil
Math Journal or Notebook*

2. **Connect** Now look at the answers to sums of consecutive triangular numbers. Do you see any connection between the answers and one of the lists of numbers?

Look at the two pictures below. They show how we can have the sums $1 + 3 = 4$ and $3 + 6 = 9$ in pictures.



3. Using two different colors, draw a picture like the ones above representing the sum $6 + 10 = 16$.

Standards: Numbers, geometry, patterns

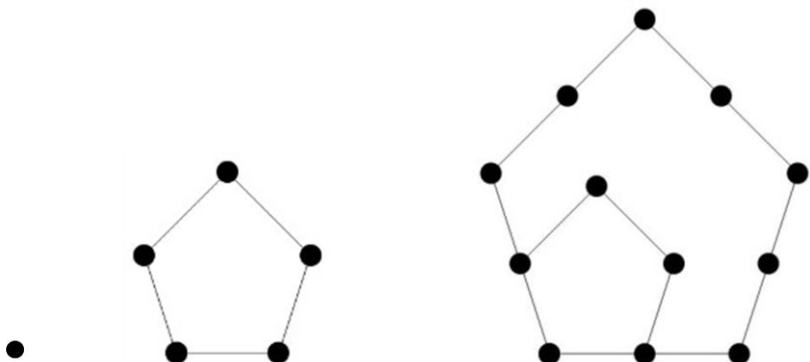
Polygonal Numbers

Lesson 6 of 6, work individually or in pairs

Materials Needed: Pencil
Math Journal or Notebook

You don't have to do just triangular numbers or square numbers! You can also do numbers that are in the shape of a pentagon or hexagon or any figure.

1. The first three pentagonal numbers are 1, 5, and 12. They are shown below. What is the next one?



2. Every pentagonal number can be written as a sum of a triangular number and a square number.

For example, 12 is a pentagonal number, $3 + 9 = 12$. 3 is a triangular number and 9 is a square number. 51 is a pentagonal number.

Which triangular number and a square number do you have to choose so that they add to 51?

3. **Extend** Choose another shape to make numbers from. Describe it in words and pictures. List the first several numbers in your number sequence. See if you can find any patterns or a general way of finding the next number in your sequence.

Standards: Numbers, patterns