Teacher Guide for Polygonal Numbers

Learning Objectives

Students will:

- Describe triangular and square numbers in terms of geometric pictures and arithmetic operations.
- Recognize patterns in triangular numbers and square numbers
- Create their own sequence (list) of polygonal numbers

Materials

- Math journal or notebook

Helping Questions

How might you begin?
What pattern do you see?
What is the next step (in the process, in the pattern)? Does that help you see any patterns?
What do you notice? What else do you notice?
Have you asked one of your classmates if he or she could see a pattern?
Why do you think that happens? (*Students may not know the answer to this question, but it helps them to think about it and try to explain it.*)
What did you try that didn't work? Can you learn something from that?
Can you explain it in a different way?

Assessment Options

- Look at the students' math journals. Ensure that they write down enough details so that they could look back and understand their work without having the lesson cards available.
- Students should be able to describe in words what they are doing in equations.

Mathematical notes

In Lesson 3, students may be tempted to try to find the "area" of the triangle by multiplying the base by the height and dividing by 2. This doesn't work because if you are counting dots, you are not counting the length and height of each triangle.

In Lesson 6, question 2 asks the students to think about how adding a square number and a triangular number makes a pentagonal number. The answer can be visualized, as you can see in the answer to question (7) on http://letsplaymath.net/2008/04/15/answer-figurate-number-puzzles/. The original questions and the story that goes with that website is at http://letsplaymath.net/2008/04/14/puzzle-figurate-numbers/.

Extensions

Students who enjoy this unit may also enjoy solving the puzzles at http://letsplaymath.net/2008/04/14/puzzle-figurate-numbers/. Some of the exercises there are similar to some of the exercises in these lessons.

Lesson 6 has an exercise for students to create their own polygonal numbers. If they have trouble getting started, you can suggest that they use hexagons or rectangles. (Try to get the students to figure this out on their own, though.) The students may come up with other kinds of polygonal numbers. As long as they're able to explain and draw them, this is good mathematics!

Also in lesson 6, you could have students look up their number pattern on the Online Encyclopedia of Integer Sequences (https://oeis.org/). It is likely that most of the results will have explanations beyond the students' ability (and perhaps beyond the ability of many professional mathematicians!), but the students may enjoy learning that their sequence is a known sequence, or that they have found something new.

Teacher Reflection

- Which students were able to see the patterns quickly, and which students struggled to see the patterns?
- Did some students see the algebraic patterns more quickly than the visual patterns, or vice versa? What extensions might be appropriate for those students next? What support can you provide for their areas of weakness?
- Did some students get frustrated if they couldn't find a pattern immediately? What can you do to ease that frustration?
- Did some students work better individually or in pairs/small groups? What support can you provide to the students to work outside their comfort zone? Or, is it better to let those students work in the group size that they find most comfortable?
- What were the greatest challenges for the students?

Standards Addressed

Common Core State Standards (and Colorado Academic Standards in Mathematics)

1. Number Sense, Properties, and Operations
2. Patterns, Functions, and Algebraic Structures
4. Shape, Dimension, and Geometric Relationship

NCTM (National Council of Teachers of Mathematics) Content Standards

Number and Operations
Algebra
Geometry

NCTM Process Standards

Problem Solving
Reasoning and Proof
Communication
Connections
Representation

Reference used

Let's Play Math, "Puzzle: Figuring out Figurate Numbers," http://letsplaymath.net/2008/04/14/puzzle-figurate-numbers/, accessed April 17, 2012.

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